Q1

1

Complete the table.

Degrees	Radians	sin	cos	tan
$30°$	$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$
135°	$\frac{3\pi}{4}$	$\frac{1}{\sqrt{2}}$	$-\frac{1}{\sqrt{2}}$	-1
180°	π	0	-1	0
$330°$	$\frac{11\pi}{6}$	$-\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$-\frac{1}{\sqrt{3}}$

[4]

$180° = \pi$ radians, so

$$(\text{degrees}) \times \frac{\pi}{180} = (\text{radians})$$

$$(\text{radians}) \times \frac{180}{\pi} = (\text{degrees})$$

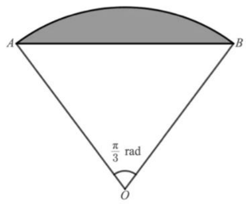
And don't forget,

$$\tan\theta \equiv \frac{\sin\theta}{\cos\theta}$$

Q2

2

The canopy of a parachute and the outermost suspension lines (cords) form a sector of a circle as shown in the diagram below, with the parachutist modelled as a particle at point O.



Not to scale

The area of the sector OAB is $\frac{49\pi}{150}$ m^2.
The parachute is made with 40 equal length suspension lines.

Disregarding any additional lengths of cord that might be required for knots, fastenings, etc., find the total length of the suspension lines required to make the parachute.

[4]

Arc length : $l = r\theta$

Area of sector : $A = \frac{1}{2}r^2\theta$

θ must be in radians!

$$\frac{49\pi}{150} = \frac{1}{2}r^2\left(\frac{\pi}{3}\right)$$

$$\frac{\cancel{\pi}}{6}r^2 = \frac{49\cancel{\pi}}{150}$$

$$\frac{1}{6}r^2 = \frac{49}{150}$$

$$r^2 = \frac{49}{25}$$

$$r = \frac{7}{5} \text{ metres}$$

Total length of suspension lines required is

$$\frac{7}{5} \times 40 = \boxed{56 \text{ metres}}$$

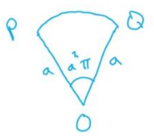
Q3

3

A sector of a circle, OPQ, is such that it has radius a cm and the angle at its centre, O, is $a^2\pi$ radians.

Given that the area of the sector OPQ in cm^2 is three times the length of the arc PQ in cm, find the value of a.

[4]



You don't have to draw a sketch to get the marks, but it can be very helpful!

Arc length : $l = r\theta$

Area of sector : $A = \frac{1}{2}r^2\theta$

θ must be in radians!

$$\frac{1}{2}r^2\theta = 3(r\theta)$$

$$\frac{1}{2}a^2(a^2\pi) = 3a(a^2\pi)$$

$$\frac{1}{2}a^4\cancel{\pi} = 3a^3\cancel{\pi}$$

$$\frac{1}{2}a^4 = 3a^3$$

$$a^4 - 6a^3 = 0$$

$$a^3(a-6) = 0$$

$$a = 0 \quad \text{or} \quad a = 6$$

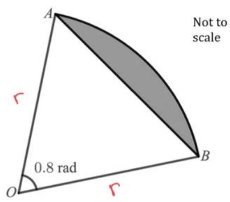
Since a is the radius of the sector, we can reject $a = 0$

$$\boxed{a = 6}$$

Q4a

4a

The diagram below shows the sector of a circle OAB.



(a) Given that the area of triangle $OAB = 5.64\ \text{cm}^2$, find the area of the shaded segment. Give your answer correct to 3 significant figures.

[5]

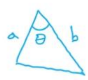
(b) Find the perimeter of the sector OAB, giving your answer correct to 3 significant figures.

[3]

Arc length : $l = r\theta$

Area of sector : $A = \frac{1}{2}r^2\theta$

θ must be in radians!



Area $= \frac{1}{2}ab\sin\theta$

a) $\frac{1}{2}(r)(r)\sin(0.8) = 5.64$

$$\frac{1}{2}r^2\sin(0.8) = 5.64$$

$$r^2 = \frac{2(5.64)}{\sin(0.8)} = 15.72440\ldots$$

The area of the shaded segment is

$$\frac{1}{2}r^2\theta - 5.64$$

$$= \frac{1}{2}(15.72440\ldots)(0.8) - 5.64$$

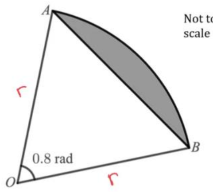
$$= 0.64976\ldots$$

$$\boxed{= 0.650\ \text{cm}^2\ (3\text{ s.f.})}$$

Q4b

4b

The diagram below shows the sector of a circle OAB.



Not to scale

(a) Given that the area of triangle $OAB = 5.64\ \text{cm}^2$, find the area of the shaded segment. Give your answer correct to 3 significant figures.

[5]

(b) Find the perimeter of the sector OAB, giving your answer correct to 3 significant figures.

[3]

Arc length : $l = r\theta$

Area of sector : $A = \frac{1}{2} r^2 \theta$

θ must be in radians!

From part (a), we know that

$$r^2 = \frac{2(5.64)}{\sin(0.8)} = 15.72440\ldots$$

b) $r = \sqrt{15.72440\ldots} = 3.96540\ldots$ cm

Perimeter $= 2r + r\theta$

$= 2(3.96540\ldots) + (3.96540\ldots)(0.8)$

$= 11.10312\ldots$

$\boxed{= 11.1 \text{ cm } (3 \text{ s.f.})}$

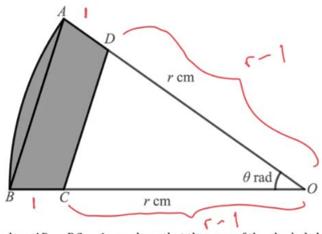
Q5a

5a

The circle sector OAB is shown in the diagram below.

The angle at the centre is θ radians, and the radii OA and OB are each equal to r cm.

Additionally, CD is parallel to AB, so that $AD = BC$ and $OD = OC$.



(a) In the case when $AD = BC = 1$ cm, show that the area of the shaded shape $ABCD$ is given by $\frac{1}{2}\theta r^2 - \frac{1}{2}(r-1)^2 \sin\theta$.

[4]

(b) Show that for small values of θ, the area of $ABCD$ is approximately $\frac{1}{2}\theta(2r-1)$.

[3]

Arc length : $l = r\theta$

Area of sector : $A = \frac{1}{2} r^2 \theta$

θ must be in radians!

a, b, θ Area $= \frac{1}{2} ab \sin\theta$

a) area of shaded shape ABCD

$= \left(\text{area of sector OAB}\right) - \left(\text{area of triangle OCD}\right)$

$= \frac{1}{2} r^2 \theta - \frac{1}{2}(r-1)(r-1)\sin\theta$

$= \frac{1}{2}\theta r^2 - \frac{1}{2}(r-1)^2 \sin\theta$

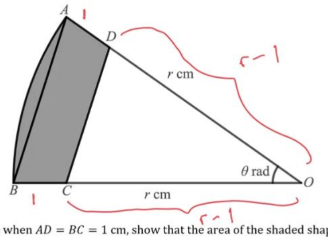
Q5b

5b

The circle sector OAB is shown in the diagram below.

The angle at the centre is θ radians, and the radii OA and OB are each equal to r cm.

Additionally, CD is parallel to AB, so that $AD = BC$ and $OD = OC$.



(a) In the case when $AD = BC = 1$ cm, show that the area of the shaded shape $ABCD$ is given by $\frac{1}{2}\theta r^2 - \frac{1}{2}(r-1)^2 \sin\theta$.

[4]

(b) Show that for small values of θ, the area of $ABCD$ is approximately $\frac{1}{2}\theta(2r-1)$.

[3]

Small angle approximations

If θ is 'small' (close to zero) and measured in radians, then

$\sin\theta \approx \theta \quad \cos\theta \approx 1 - \frac{1}{2}\theta^2 \quad \tan\theta \approx \theta$

b) Since θ is small and measured in radians,

$$\frac{1}{2}\theta r^2 - \frac{1}{2}(r-1)^2 \sin\theta \approx \frac{1}{2}\theta r^2 - \frac{1}{2}(r-1)^2\theta$$

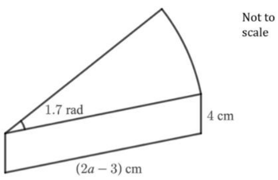
$$= \frac{1}{2}\theta\left(r^2 - (r-1)^2\right)$$

$$= \frac{1}{2}\theta\left(r^2 - (r^2 - 2r + 1)\right)$$

$$\boxed{= \frac{1}{2}\theta(2r-1)}$$

Q6

6



The diagram shows a prism with cross-section in the shape of a sector of a circle.

The radius of the sector is $(2a-3)$ cm, and the angle at the centre is 1.7 radians.

The height of the prism is 4 cm.

Given that the volume of the prism is 7.65 cm^3, find the possible value(s) of a.

[5]

Arc length : $l = r\theta$

Area of sector : $A = \frac{1}{2}r^2\theta$

θ must be in radians!

Volume of prism = (area of cross-section) × (height)

$$\text{Volume} = \left(\frac{1}{2}(2a-3)^2(1.7)\right) \times (4) = 7.65\text{ cm}^3$$

So,

$$3.4(2a-3)^2 = 7.65$$

$$(2a-3)^2 = 2.25$$

So $2a - 3 = 1.5$ or -1.5

But $2a-3$ is a radius and cannot be negative.

$$2a - 3 = 1.5$$

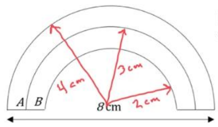
$$2a = 4.5$$

$$\boxed{a = 2.25}$$

Q7

7

A rainbow-shaped logo is formed from three semicircles as shown below.
Each of the inner semicircles has a radius that is 1 cm less than that of the next semicircle further out.



Not to scale

Find, in simplest terms, the ratio of the outer area A of the logo, to the inner area B.

[5]

Arc length : $l = r\theta$

Area of sector : $A = \frac{1}{2} r^2 \theta$

θ must be in radians!

And note : A semicircle is a sector of a circle where the angle at the centre, θ, is π radians (180°)

area of semicircle $= \frac{1}{2}\pi r^2$

area of A $= \frac{1}{2}\pi(4)^2 - \frac{1}{2}\pi(3)^2 = \frac{7}{2}\pi$

area of B $= \frac{1}{2}\pi(3)^2 - \frac{1}{2}\pi(2)^2 = \frac{5}{2}\pi$

So the ratio is

$\frac{7}{2}\pi : \frac{5}{2}\pi$

$= 7\cancel{\pi} : 5\cancel{\pi}$

$\boxed{= 7 : 5}$